EXPERIMENTAL GEOMETRY*

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Before beginning a discussion of the actual topic assigned for this paper, let us consider for a short time what are the requirements that determine whether or not a subject should have a place in the school curriculum. It does seem that we have emphasized two things entirely too much. These are, the ease of handling the subject on the part of pupil and teacher and the practical value of the subject. These two should be considered of course, but there are other considerations of more importance.

The two main objectives of school work are the development of character and the training of the mind and any subject which does not assist in these two objectives better than any other which may be substituted for it is unworthy a place in the course of study of any school. We will except, however, such subjects as stenography, printing, etc., when pursued for the purpose of preparing the individual for a definite occupation. A third objective is the practical value of the subject. This must not be overlooked. If we can give the pupil some experiences in school similar to those that he will encounter in later life, we are gaining much. But on the other hand, there is danger of overemphasizing this third objective. Even Manual Training when taught in the usual way has more value in the development of character and in formal discipline than it has as practical worth. The boy who learns to make a useful article at the manual training bench will cultivate habits of neatness and thoroughness that will enable him to do better work elsewhere.

Now, how does Mathematics stand the test of the preceding paragraphs? In the first place it is a study of the unchangeable laws of the universe. It is certainly true that the fundamental laws of science and mathematics, with their marvelous and intricate dependence one upon another are of more value to the individual than the principles of a language he will never speak. The laws of Mathematics and Science are God-made,

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while the principles of language and history are due to man's intelligence.

A proper study of Mathematics assists in the development of character. The pupil soon learns that, to get correct results in mathematical processes, he must obey the laws of the subject which he has learned and must obey them implicitly. A failure to reach accurate conclusions is due to his own errors and nothing else. What this experience is worth to the individual in later years is difficult to estimate, but since nearly all the actual failures in life are due to the individual's mistakes and negligence and not to outside influences, it is well for him to encounter early that study in which he pays dearly for his own mistakes.

Furthermore, the difficult problems of Mathematics furnish abundant material to test the powers of the keenest minds, while the weaker student may at the same time find exercises within the range of his ability. One of the most valuable results from the study of mathematics is the acquiring of a good habit of preparing the day's lesson. The assignments of Algebra and Geometry are definite enough that even the student who has never learned how to study is soon able to prepare a portion of each lesson and to know positively that that much of the lesson has been completed. As he becomes more and more able to do all of the day's assignments, he develops a habit of study that will benefit him during his whole life.

But let us come nearer home to our subject and ask the definite question, "Why Study Geometry?" In what way does this subject aid in the development of the individual so that it is entitled to a place in the school curriculum? Is it more important than other subjects of Mathematics and Science that might be put in its place? Let us answer these questions from the standpoint of what has been said in the preceding paragraphs. Geometry in common with all mathematics deals with the fundamental facts of the universe and every person of culture and refinement should know something of these truths. The poetry has never been written, the music has never been played that contains one-tenth the appeal to the human soul that is found in the harmonious motion of planets, stars, and other heavenly bodies. The difficulty, however, is that many of the

beauties of creation are beyond the comprehension of the average mind on account of the inability to understand the mathematical principles on which these marvelous things depend.

I feel that I owe you an apology for using so much time on this part of my paper on matter that is not immediately connected with the topic assigned me, but I feel that I could not consistently discuss the topic of experimental Geometry without putting it on the same basis as any other branch of mathematics. Certainly we all want to feel that there is more worth in the subject than merely teaching a new method for drawing a right angle or for ripping a board into several equal strips.

There are three ways for the student to get possession of the truths of Geometry. The first method is by experiment, as, for example, he constructs several equilateral triangles and, after measuring the angles, he arrives at the conclusion that the value of each angle of an equilateral triangle is 60°. While such reasoning is not logically sound, yet we must admit that many of the facts of science have been discovered in just this way.

The second method is by experiment followed by formal proof. This way is the most logical of all. For example the student finds that the two diagonals of the rectangle that he has constructed are equal. After measuring the diagonal of other rectangles, he proceeds to prove that the diagonals of any rectangle are equal.

The third method is by logical proof without experimentation. The student works with given propositions to discover their proofs or he attempts to discover new truths by juggling logically known facts. I think we will agree that this third method is the most difficult of all. We have wasted much time in the past by beginning Geometry at this point. Even though the student has no experience with geometry till the tenth year, yet it would be wise to devote some time, even a half year, to the first two methods of acquiring mathematical truths, before we require formal proofs. This, however, is merely a makeshift to take care of those students who arrive at the tenth year without any knowledge of Geometry. The young student beginning the study of Geometry, in the latter years of the grades or the early years of the junior high school, should learn at first

a few definitions and then proceed to the use of the tools of the subject. The necessary vocabulary after the first definitions will be obtained gradually as the need arises. All of the work should be centered about the construction processes. He will need and should learn to use the following tools and materials:

A hardwood straight-edge divided into inches and centimeters; a pair of compasses; protractor; 30°, 60°, 90° right triangle; 45°, 45°, 90° right triangle; good drawing paper, cross-section paper, and good firm paper for experiments in paper folding, and a supply of yard-sticks, string, wire, thumb-tacks, etc.

The work may proceed as follows: After the student has become familiar with the use of the tools he should learn the simpler constructions with the ruler and compasses, such as bisecting a line segment, constructing a perpendicular to a line at any point on the line or through any point not on the line, bisecting an angle, and constructing an angle equal to a given angle. After these constructions are mastered and the pupil has tested his results in many ways he is ready to settle down to definite experimental study.

The work should all be done by means of the laboratory plan. He must be given little hint of what the results are to be but must find them out for himself. As, for example, one day's lesson is about the value of the angles of triangles. We will assume that he has already learned how to construct a triangle with the compasses and straight-edge when the lengths of the three sides are given.

The pupil will do the following exercises:

(a) Draw several triangles and measure the angles. Find the sum of the angles of each triangle. What is your conclusion?
(b) Construct several isosceles triangles and measure the angles. What relation do you find between any of the angles of an isosceles triangle?
(c) Construct an equilateral triangle and measure the angles. What is your conclusion.
(d) Darw several scalene triangles and measure the angles. Where is the largest angle in each of the scalene triangles?
(e) Can you find the sum of the two acute angles of a right triangle without measuring?

After the young pupil has done a considerable amount of experimental work, he can begin simple proofs in an informal way. The five well-known theorems relating to the congruence

of triangles should be done entirely by experiment. After the student is satisfied that they are true he can undertake such simple exercises as the following:

(a) *ABC* and *XYZ* are two congruent triangles, angle *A* equal to angle *X*, etc. The altitudes *CD* and *ZK* are drawn to the respective bases *AB* and *XY*. Draw the two figures and show the triangles *ADC* and *XKZ* congruent.
(b) Draw a line from the vertex of an isosceles triangle to the middle point of the base and show the two triangles congruent.
(c) *AB* and *CD* are two equal chords in a circle whose center is *O*. Draw *OA, OB, OC* and *OD* and show the two triangles to be congruent.

The amount of material available for experimental work in Geometry is so great that no class can be expected to do a great part of it. The teacher should make a careful selection from the following:

(a) Study of lines, angles, triangles, polygons and circles.
(b) Parallels.
(c) Drawing to scale and similar figures
(d) Accurate proportions with the bar graph and circle graph.
(e) Symmetry.
(f) Ratio and proportion.
(g) Constructing by paper folding perpendiculars, bisectors of angles, bisectors of lines, etc.
(h) Pythagorean theorem.

It does not seem wise to say positively where the material already discussed should be done. Certainly some of it should be done before the end of the eighth year. In schools where the mathematical program can be so arranged, a considerable amount of work in experimental Geometry can be done in the seventh year. Of course, if it is done in the seventh year, the work must be simplified to be within the range of ability of the seventh grade pupil. If experimental Geometry is neglected entirely till the tenth year, then by all means give a few weeks of it as an introduction to formal Geometry.

So far we have dodged the question of unified mathematics. Experimental geometry will be greatly enriched by teaching something of Algebra at the same time. Many of the formulas of Algebra can be illustrated so nicely with the rectangle, bar graph, etc., that it seems a shame if the pupil does not have this correlation of mathematics.

We will close this paper with a few suggested exercises for experimental geometry. It will be noticed that these exercises are in groups and the last one in the group, if possible, deals with the practical application. They are arranged in this way merely to add interest to the subject, and thereby to make it more attractive. At the same time let us use all the means in our power to convince the pupil that the subject of Geometry contains much more than the practical application.

Suggested Exercises.

1. Construct a rectangle and draw the diagonals. Measure the lengths of the four segments into which the two diagonals are divided at the point of intersection. Where is the center of the rectangle? How can you find the point in the center of the ceiling of a room where a light fixture should be placed?

2. Construct several perpendiculars to the same line at different points on the line. What do you notice about these perpendiculars? How can you use your right triangle to draw a number of parallel lines on a sheet of paper?

3. Make a quadrilateral out of four yard-sticks by fastening them together at the vertices with a single nail at each vertex. Can you change the shape of the quadrilateral? Is it always a parallelogram? A square? Measure the diagonals. Are they equal? Write several conclusions. Could you use this instrument to draw parallel lines on the blackboard?

4. Draw any triangle. Construct a second triangle with sides twice as long as the sides of the first triangle, and construct a third triangle with sides three times as long as the sides of the firsttriangle. What do you notice about the shape of the triangles? Measure the corresponding angles of the triangles. State your discovery as a theorem.

5. Draw a circle. Construct a central angle of 90°. Draw several inscribed angles intercepting the same arc that is intercepted by the central right angle. Measure the inscribed angles with the protractor. Repeat the process with a central angle of 120°. State your conclusion as a theorem.

6. Draw a rectangle. Draw a second rectangle upon a base equal to that of the first rectangle whose area is three times as great. How do their altitudes compare. The areas of rectangles having equal bases have what ratio?

7. Draw a triangle with one side twice as long as one of the other sides. Bisect the angle included by these sides. Measure the segments into which the bisector divides the opposite sides. What is their ratio? How does it compare with the ratio of the two sides of the angle bisected? State the fact in the form of a theorem.

8. Draw a triangle. Upon a base three times as long as the base of the first triangle draw a second triangle similar to the first. Draw the altittudes to the bases. How do the altitudes compare in length? Compare the areas. Tell how to construct a triangle similar to a given triangle with four times the area.

9. A man whose incomes is $3,000 per year spends $500 for rent, $800 for provisions, $300 for fuel and other household expenses, $600 for clothing, $200 for charity, $300 for luxuries, and save the remainder.

Construct a circle graph to show the various expenditudes. Construct bar graphs to show the same expenditures, using one square centimeter for each $100. Which do you consider the getter graph.

10. Draw any triangle. Construct and measure one altitude, measure the base to which the altitude is drawn, and compute the area of the triangle. Draw any quadrilateral. Draw a diagonal dividing it into two triangles and by the preceding method compute the area of the two triangles. Explain how you could find the area of a four sided lot that is not a rectangle.

11. The three sides of a triangular lot are 18, 20 and 24 rods. Draw a triangle to the scale of one centimeter to one rod and compute the area of the triangle and the lot.

12. Draw any polygon. Extend one side at each vertex. Beginning with one exterior angle rotate a pencil through all of the exterior angles. What is the amount of the rotation? Draw another polygon having a different number of sides. Repeat the experiment. What do you conclude about the sum of the exterior angles of a polygon?